Math 242 Class Exercise 3: Gaussian Elimination, Row Echelon Form, and Reduced Row Echelon Form

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Elementary Row Operations. The following operations on an $m \times n$ matrix are called **elementary row operations**:

1. Multiply any row by a non-zero number.
2. Interchange any two rows.
3. Add a multiple of one row to another.

Word of caution: Do not make up row operations. Doing so will change the structure of the linear system and hence the solution. i.e. incorrect solution.

Row Echelon Form (REF) An $m \times n$ matrix is in **row echelon form** if it meets all of the following criteria:

1. All nonzero rows are above any rows of all zeros.
2. Each leading entry of a row is in a column to the right of the leading entry of the row above it.
3. All entries in a column below a leading entry are zeros

Note, that for REF, some authors add the condition that each leading entry must be a 1 which is called a *leading 1*.

Reduced Row Echelon Form An $m \times n$ matrix is in **reduced row echelon form** if it meets all of the following criteria:

1. It is in Row Echelon Form (REF).
2. The leading entry in each nonzero row is a 1. (called a *leading 1*).
3. Each leading 1 is the only non-zero entry in its column.

Exercises:

1. Create 4 matrices where two matrices are in REF, one is in RREF, and one is in neither form. Do not label what they are. Hand them to a partner and have them try to decide in 3 mins time what form each matrix is in. After the 3 mins time is up, discuss and reach consensus on the final answers.
2. Create an augmented matrix associated to the system below. Put the matrix in REF and then use back substitution to find the solution. After you are done, put the matrix in RREF and then write out the solution.

$$\begin{aligned} x + y + 5z + 2w &= -1 \\ y + 3z - w &= 1 \\ 2x + 4z + w &= 1 \end{aligned}$$

3. Create an augmented matrix associated to the system below. Put the matrix in REF and then use back substitution to find the solution. After you are done, put the matrix in RREF and then write out the solution.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= -2 \\ 2x_1 + 5x_2 + 5x_3 &= -3 \\ x_1 + 3x_2 + 2x_3 &= -1 \end{aligned}$$

4. Create an augmented matrix associated to the system below. Put the matrix in REF and then use back substitution to find the solution. After you are done, put the matrix in RREF and then write out the solution.

$$\begin{aligned} 4x_1 - 2x_2 - 3x_3 + x_4 &= 3 \\ 2x_1 - 2x_2 - 5x_3 &= -10 \\ 4x_1 + x_2 + 2x_3 + x_4 &= 17 \\ 3x_1 + x_3 + x_4 &= 12 \end{aligned}$$

5. What are the advantages/disadvantages of using REF + Back Substitution vs using RREF?

6. Are REF's and RREF's unique?