

Math 242 Class Exercise 6:

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Exercises:

1. Create 2 vectors \vec{v}_1 and \vec{v}_2 in \mathbf{R}^2 and describe the $\text{span}\{\vec{v}_1, \vec{v}_2\}$ both algebraically and geometrically. Do the same for \mathbf{R}^3 .
2. For the following system, can you rewrite it in the form $\mathbf{Ax} = \mathbf{b}$? Here, \mathbf{A} is a matrix and \mathbf{x} is a vector. How can you define the matrix vector product?

$$\begin{aligned}x_1 + x_2 + x_3 + x_4 &= 1 \\x_1 + 2x_2 + 4x_3 + 4x_4 &= 1 \\x_1 + 3x_2 + 3x_3 + 3x_4 &= 1 \\x_1 + 4x_2 + x_3 + 2x_4 &= 2\end{aligned}\tag{1}$$

3. From problem 2, can you write $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4]$ where the \mathbf{a}_i are column vectors?
4. What can you deduce about the relationship between the augmented matrix

$$[\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4 | \mathbf{b}]$$

and the solution to the above system (1).

5. What does it mean for two vectors in general to be linearly independent? Can you describe this both algebraically and geometrically?
6. A mining company has 2 mines. One day's operation at mine #1 produces ore that contains 30 metric tons of copper and 600 kilograms of silver, while one day's operation at mine #2 produces ore that contains 40 metric tons of copper and 380 kilograms of silver. Let \vec{v}_1 and \vec{v}_2 be the following:

$$\vec{v}_1 = \begin{bmatrix} 30 \\ 600 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 40 \\ 380 \end{bmatrix}$$

Then \vec{v}_1 and \vec{v}_2 represent the output of mine #1 and #2 respectively.

- (a) What physical interpretation can be given to $5\vec{v}_1$?
- (b) Suppose the company operates mine #1 for x_1 days and mine #2 for x_2 days. Write a vector equation whose solution gives the number of days each mine should operate in order to produce 240 tons of copper and 2824 kilograms of silver. Use whatever means you have to solve this problem.
- (c) Suppose now that both mine #1 and mine #2 now produce gold. If the daily output of gold for mine #1 is 375 kilograms of gold and for mine #2 150 kilograms of gold. How many days does each mine need to operate in order to produce 240 tons of copper, 2824 kilograms of silver, and 1480 kilograms of gold? Use whatever means you have to solve this problem.
- (d) How many days should each mine operate in order to produce 750 tons of copper and 7000 kilograms of silver and 3700 kilograms of gold? Use whatever means you have to solve this problem.