Math 79 HW #4

~~The book section is in HW#3~~

1. Section 12.1: 2,4,5,6, 8,10,12,20,34
2. Section 12.2: 5,7,8

3. For a population of people living on an island, the number present after m years is given by $(1.45)^m \times 8500$. Find each of the following.
   a. The number of people initially on the island
   b. The annual growth rate
   c. The total island population after 2 years
   d. The total island population after 15 years

4. For a fast-growing population of mammals, the number present after m years is given by $(1.75)^m \times 50$. Find each of the following.
   a. The initial number of mammals
   b. The annual rate of growth
   c. The total mammal population after 3 years
   d. The total mammal population after 20 years

5. Suppose a population grows at the rate of 3% each year. If the initial population is 50,000, what is the population after 5 years? 20 years? 45 years?

6. Suppose a population grows at the rate of 4% each year. If the initial population is 75,000, what is the population after 12 years? 50 years? 100 years?

7. The population of the city of Limon, Costa Rica, grew from 168,000 in 1984 to 380,000 in 2004. Assuming a Malthusian population model, find the annual rate of growth.

8. The population of the city of Gifu, Japan, grew from 2,029,000 in 1985 to 2,112,100 in 2004. Assuming a Malthusian population model, find the annual rate of growth.

9. In 1995, two "sister" cities each had populations of approximately 35,000. Their annual growth rates differed, however. One city had an annual growth rate of 2%, while the other had an annual growth rate of 8%. Assuming that a Malthusian population model applies to these cities, predict the difference in their populations in 2005.

10. In the year 2000, city A had a population of 20,000 and city B had a population of 25,000. If their annual growth rates are 4.5% and 2.25%, respectively, in what year would you predict that the population of city A will exceed that of city B? Assume a Malthusian population model applies to these cities. Use trial and error.

11. In 1990, the population of the United States was approximately 249 million and was increasing at a rate of 0.7% per year. Assuming the growth rate remains the same, what is the anticipated size of the U.S. population in the years 2005 and 2010? Round to the nearest million.

12. In 2000, the population of the United States was approximately 281 million and was increasing at a rate of 1.1% per year. Assuming the growth rate remains the same, what is the anticipated size of the U.S. population in the years 2010 and 2015? Round to the nearest million.

13. In 2003, the population of Spain was approximately 40 million and was increasing at a rate of 0.16% per year.
   a. If the growth rate remains the same, predict the population of Spain in 2020. Round to the nearest thousand.
   b. If the growth rate from 2003 through the year 2010 is 0.16%, but the growth rate increases to 0.32% for the year 2011 and remains constant at 0.32% from 2011 through 2020, then predict the population in the year 2020. Round to the nearest thousand.

14. In 2000, the population of the United States was approximately 281 million and was increasing at a rate of 1.1% per year.
   a. If the growth rate remains the same, predict the population of the United States in 2020. Round to the nearest thousand.
   b. If the growth rate remains 1.1% through the year 2010, but decreases to 0.55% in 2011 and remains constant from 2011 to 2020, predict the population in the year 2020. Round to the nearest thousand.

15. In 1626, Peter Minuit of the Dutch East India Company purchased the island of Manhattan for the equivalent of $24 in trading goods. What would the value of these goods be in 2005 if their value had grown by a constant annual rate of 3% since that time? What would their value be if the annual rate of growth were 4%? Round to the nearest dollar.

16. Suppose you purchased a home in 1993 for $80,000. A review of real estate records for the past 10 years indicates that the average value of houses in your area has increased by an average of 4.5% per year. If this rate of growth continues, how much should the house be worth in 2007? Round to the nearest dollar.

17. The bacteria *E. coli* duplicates itself approximately every 20 minutes under ideal circumstances.

a. Fill in the following table by considering 20-minute intervals of time. Begin with one *E. coli* bacterium.

20-Minute Time Interval m	Total Number of *E. coli*
0	1
1	
2	
3	
4	

b. Use the formula for the growth rate to find the value of r in this case, where m is measured in 20-minute intervals of time.

c. Assuming that the rate of growth remains constant, write the Malthusian population growth formula of the form $P_m = (1 + r)^m \times P_0$. Use the rate you found in part (b) and let m represent the number of 20-minute time intervals.

d. Use your model from part (c) to find the total number of *E. coli* present after 4 hours and then after 1 day. Remember that time is measured in 20-minute intervals.

18. Under less-than-ideal conditions, such as lacking sufficient nutrients, *Nanobacterium sanguineum* can slow its growth rate so that it duplicates itself once every 6 days.

a. Fill in the following table by considering 6-day periods of time. Begin with one bacterium.

6-Day Time Interval m	Total Number of *N. sanguineum*
0	1
1	
2	
3	
4	

b. Use the formula for the growth rate to find the value of r in this case, where m is measured in 6-day intervals of time.

c. Assuming that the rate of growth remains constant, write the Malthusian population growth formula of the form $P_m = (1 + r)^m \times P_0$. Use the rate you found in part (b) and let m represent 6-day time intervals.

d. Use your model from part (c) to find the total number of *N. sanguineum* present after 33 days and then after 7 weeks. Remember that time is measured in 6-day intervals.

19. Your home appreciated from $145,000 to $215,000 in 5 years.

a. Find the annual rate of growth in the value of the house.

b. Assuming that a Malthusian population model applies to the value of your home, predict the value of your home after 5 more years.

20. A population of apes increased from 24 to 55 in 2 years.

a. Find the annual rate of growth in the ape population.

b. Assuming that a Malthusian population model applies to the ape population, predict the total number of apes after 2 more years.

21. The population of a city grew from 20,000 to 33,000 during the past 10 years. If growth continues at this rate, what will the population be in 20 more years?

22. The population of a given city grew from 35,000 to 42,000 during the past 15 years. If growth continues at this rate, what will the population be in 17 more years?

23. During the past 8 years, the population of Greenville grew from 28,000 to 35,500. If growth continues at this rate, what do you predict the population will be in 22 more years?

24. Between 1985 and 1993, the population of Braxton grew from 62,400 to 70,800. If this rate of growth continues, what do you predict the population will be in 2010?

25. According to Statistics Canada, the population estimates for Ontario in the years 1999 to 2002 were as follows: 11,527,900; 11,697,600; 11,894,900; and 12,068,300.

a. Calculate the annual rate of growth for each pair of consecutive years.

b. Can a Malthusian population model be assumed for this population? Explain why or why not.

26. According to Statistics Canada, the population estimates for British Columbia in the years 1999 to 2002 were as follows: 4,028,300; 4,060,100; 4,101,600; and 4,141,300.

a. Calculate the annual rate of growth for each pair of consecutive years.

b. Can a Malthusian population model be assumed for this population? Explain why or why not.

27. The world's population increased from about 4.4 billion in 1980 to about 6.3 billion in 2003.

a. Assuming the growth rate was constant, find the annual rate of growth for the world's population.

b. Use a Malthusian population model to predict the population of the world in the year 2010. Round to the nearest tenth of a billion.

c. There is concern that the world is overpopulated. Consider the total land area of Rhode Island, the smallest state, which is approximately 1045 square miles. If each of the world's 6.3 billion people stood on a 2-foot-by-2-foot square, how many square miles would be needed? (Recall that 5280 feet = 1 mile.) How many states the size of Rhode Island would this use?

28. In August 2004, the world's population was 6.38 billion. Assume the population grows at a constant annual rate of 1.13%.

a. Write the Malthusian population growth formula of the form $P_n = (1 + r)^n \times P_0$ that describes the world population growth.

b. The total land area of Alaska, the largest state, is approximately 586,412 square miles. If each person in the world stood on a 2-foot-by-2-foot square, in how many years would the world's population fill all the 2-foot-by-2-foot squares in Alaska? (Remember that 1 mile = 5280 feet.)

29. The current annual population growth rate in the United States is approximately 1.13%. If this rate remains constant, in how many years will the population of the United States double in size? Use trial and error.

30. Kenya, with an annual population growth rate of 4%, had the highest growth rate of any country in the world in 1994. Assuming that the growth rate remains constant, how long will it take Kenya's population to double? Use trial and error.

31. Suppose there are five investors in the first quarter of a Ponzi scheme. The investors are guaranteed a 40% rate of growth and time is measured in quarters of a year. What is the minimum number of people who must be investing in each of the next three quarters in order to pay off the investors from the previous quarter?

32. Suppose a Ponzi scheme has 80 investors during its first quarter. The investors are guaranteed a 40% rate of growth and time is measured in quarters of a year. What is the minimum number of people who must be investing in each quarter of the first 2 years in order to pay off the investors from the previous quarter?

33. If you started a Ponzi scheme like the one in the text (returning 40% in 90 days) with 10 investors, approximately how long would it take until every man, woman, and child in the United States would need to be an investor in order to keep the scheme going? Assume there are currently 290,000,000 people in the United States.

34. If you started a Ponzi scheme that guaranteed a return of 75% in 90 days, with 20 investors, approximately how long would it take until every man, woman, and child in the world would need to be an investor in order to keep the scheme going? Assume there are 6,280,000,000 people currently in the world.

35. Suppose a chain letter has 5 levels, asks you to send $2 to the person at the top of the list, and requires you to send out 10 new letters.

a. What is the payoff in rising from the bottom of the list to the top?

b. How many people must participate in order for an individual to rise from the bottom of the list to the top?

36. Suppose a chain letter has 6 levels, asks you to send $5 to the person at the top, and requires you to send out 4 new letters.

a. What is the payoff in rising from the bottom of the list to the top of the list?

b. How many people must participate in order for an individual to rise from the bottom of the list to the top?

37. Suppose a chain letter has 4 levels and requires you to send out 5 new letters. What are the payoffs in rising from the bottom of the list to the top of the list if you are required to pay the following amounts to the person at the top of the list?

a. \$1 **b.** \$5 **c.** \$10 **d.** \$50

38. Suppose a chain letter has 4 levels and requires you to pay \$5 to the person at the top of the list. What are the payoffs in rising from the bottom of the list to the top of the list if you are required to send out the following numbers of letters?

a. 1 **b.** 5 **c.** 10 **d.** 50

39. Suppose you want to create a chain letter that will give you a payoff of about \$90,000. You assume that you are more likely to encourage people to join if the cost is only \$3. Devise a scheme that will generate about \$90,000 for those who rise from the bottom of the list to the top.

40. Suppose you want to create a chain letter that will give you a payoff of about \$500,000. You assume that you are more likely to encourage people to join if they have to send out only 5 letters. Devise a scheme that will generate about \$500,000 for those who rise from the bottom of the list to the top.

Extended Problems

41. A sheet of paper is approximately 0.1 millimeter thick. If you fold the sheet of paper in half one time, you will have two layers, with a total thickness of approximately 0.2 millimeter. If you fold the paper in half again, you will have four layers, with a total thickness of approximately 0.4 millimeter. Create a table showing the thickness after each of the first 10 folds. If a piece of paper was big enough and you could fold it an unlimited number of times, how many folds would it take to create a thickness equal to 480 meters, which is the height of the Union Square Phase 7 building in Hong Kong?

42. The bacteria *E. coli* divides approximately every 20 minutes under ideal conditions. It measures about 2 micrometers in length and 0.8 micrometers in diameter, so it has a volume of about 1 cubic micrometer. A micrometer is one-millionth of a meter. Suppose one *E. coli* bacterium is present initially.

a. How many bacteria are present after 2 hours, and what is their total volume?

b. How many bacteria are present after 5 hours, and what is their total volume?

c. About how long would it take for the volume of cells to fill up a space equal to 1 cubic meter?

d. About how long would it take for the volume of cells to fill up a typical swimming pool, with a volume of 13,750 cubic meters?

43. *Nanobacterium sanguineum* is a unique bacteria species. It can slow or speed up its rate of growth depending on the conditions in which it finds itself. Research this interesting bacterium and write a short report. Be sure to include information about where this bacterium is found, what it does to its host, and what its strengths and weaknesses are. Include information about its different rates of growth. Create several tables to demonstrate growth patterns under different conditions and rates of growth, and explain what conditions lead to the different rates of growth. On the Internet, search keyword "Nanobacterium sanguineum."

44. The United States experienced an increase of approximately 33 million people between 1990 and 2000. However, population growth did not occur evenly throughout the country. Over the 10-year period from 1990 to 2000, the populations of some states grew much more rapidly than others. On the Internet, visit the U.S. Census Bureau website at www.census.gov, and find the populations for each state in 1990 and in 2000. Calculate the 10-year growth rate for each state. Which state(s) had the largest rates of growth? Which had the smallest rates of growth? Draw, or use the Internet to download, an outline map of the United States. Color groups of states with the greatest growth rates the same. Color groups of states with the smallest rates of growth the same. Summarize any patterns in growth rates you find in a short essay.

4. In 1961, there were approximately 50,000 prairie chickens in New Mexico. By 1979, the population had dropped to about 10,000. Assume a Malthusian population model.
   a. Find the rate of decline for the population.
   b. Using the rate found in part (a), write the equation of the form $P_m = (1 + r)^m \times P_0$ that represents the prairie chicken population after m years, where $m = 0$ in 1961. Predict the population in the years 2010 and 2031.
   c. If the population continues to decline at the same rate, in approximately what year will there be only one prairie chicken left? Use the guess-and-test method and your calculator.

5. Suppose a radioactive substance has an annual decay rate of 3%. Initially, there are 27 kilograms of the substance. Fill in the following table to show the amount of the substance left at the end of each year. Use the radioactive decay formula $A_m = (1 - d)^m \times A_0$.

Time in Years	Amount of Substance Present, in Kilograms
0	$(1 - 0.03)^0 \times 27 =$
1	$(1 - 0.03)^1 \times 27 =$
2	
3	
m	

6. Suppose a radioactive substance has an annual decay rate of 1.35%. Initially, there are 16 grams of the substance. Fill in the following table to show the amount of the substance left at the end of each year. Use the radioactive decay formula $A_m = (1 - d)^m \times A_0$.

Time in Years	Amount of Substance Present, in Grams
0	$(1 - 0.0135)^0 \times 16 =$
1	$(1 - 0.0135)^1 \times 16 =$
2	
3	
m	

7. If the amount, in grams, of a radioactive substance present after m years is given by $A_m = (0.955)^m \times 1400$, find each of the following.
   a. How much of the radioactive substance was present initially?
   b. What is the decay rate?
   c. How much of the substance had decayed after 1 year?
   d. How much of the substance remained after 20 years?
   e. How much of the substance remained after 51 years?

8. If the amount, in grams, of a radioactive substance present after m years is given by $A_m = (0.85)^m \times 500$, find each of the following.
   a. How much of the radioactive substance was present initially?
   b. What is the decay rate?
   c. How much of the substance had decayed after 1 year?
   d. How much of the substance remained after 3 year?
   e. How much of the substance remained after 4.265 years?

9. Suppose that a radioactive substance has an annual decay rate of 2.5%. If 300 grams of the substance are present initially, how much of the substance will remain after 5 years? 10 years? 15 years?

10. Suppose that a radioactive substance has an annual decay rate of 0.17%. If 5000 grams of the substance are present initially, how much of the substance will remain after 20 years? 100 years? 3500 years?

11. The half-life of strontium-90 is 28 years. What is the annual decay rate for strontium-90?

12. The half-life of uranium-233 is 160,000 years. What is the annual decay rate for uranium-233?

13. The half-life of plutonium-241 is 13 years. A sample contains 50 grams of the substance.
   a. What is the annual decay rate of plutonium-241?
   b. How much of the sample will remain after 6.5 years?
   c. How much of the sample will remain after 13 years? 26 years? 39 years?

14. The half-life of radium-226 is 1620 years. A sample contains 2500 grams of the substance.
   a. What is the annual decay rate of radium-226?
   b. How much of the sample will remain after 25,920 years?
   c. How much of the sample will remain after 810 years? 1620 years? 3240 years?

15. The half-life of sodium-22 is 2.6 years.

a. What is the annual decay rate for sodium-22?

b. If there are 200 grams of sodium-22 in the year 2000, how much will remain in the year 2035?

16. Suppose the half-life of a radioactive substance is 73 days.

a. What is the annual decay rate for this substance?

b. If 200 grams of the substance were available on January 1, 2001, how much would remain on December 3, 2001?

17. Suppose the half-life of a radioactive element is 18 minutes. If there are 10 grams of the substance initially, how much will be left 4 hours later?

18. The half-life of argon-41 is 1.8 hours. If 50 grams of the substance were available at noon, how much would remain at midnight?

19. Plutonium-241 has a half-life of 13 years. If a sample of 100 grams was produced in 1950, how much of the sample remained in 2003?

20. Suppose the half-life for a radioactive substance is 400 years. How much will remain after 2000 years if 100 grams were present initially?

Problems 21 through 24

Use the half-life radioactive decay formula

$$A_m = \left(\frac{1}{2}\right)^{\frac{m}{h}} \times A_0.$$

21. Suppose that initially 500 grams of a radioactive substance are contained in a sample, so $A_0 = 500$. For each of the following situations, calculate how much of the substance will remain after $m = 100$ years.

a. Suppose the half-life is 100 years.

b. Suppose the half-life is 50 years.

c. Suppose the half-life is 10 years.

d. Suppose the half-life is 1 year.

22. Suppose that initially 30 grams of a radioactive substance are contained in a sample, so $A_0 = 30$. For each of the following situations, calculate how much of the substance will remain after $m = 50$ days.

a. Suppose the half-life is 100 days.

b. Suppose the half-life is 50 days.

c. Suppose the half-life is 10 days.

d. Suppose the half-life is 1 day.

23. Suppose the half-life of a certain radioactive substance is 2 years, so $h = 2$. Initially there are $A_0 = 100$ grams of the substance. Calculate how much of the substance will remain after each of the following periods.

a. 1 year

b. 18 months

c. 2 years

d. 4 years

e. 8 years

24. Suppose the half-life of a certain radioactive substance is 5000 years, so $h = 5000$. Initially there are $A_0 = 100$ grams of the substance. Calculate how much of the substance will remain after each of the following periods.

a. 1 year

b. 100 years

c. 1000 years

d. 10,000 years

Problems 25 and 26

Use the half-life approximation formula:

$$h \approx \frac{0.693}{d + \frac{d^2}{2}}.$$

25. A radioactive substance has an annual decay rate of 2.445%.

a. What is the approximate half-life for this substance?

b. Compare the half-life with those given in Table 12.5. Which element could this be?

26. A radioactive substance has an annual decay rate of 0.04278%.

a. What is the approximate half-life for this substance?

b. Compare the half-life with those given in Table 12.5. Which element could this be?

27. A fossil is tested for the level of ^{14}C. It is found that the fossil contains about 90% of its original amount. Estimate the age of the fossil.

28. If 500 milligrams of ^{14}C are present in a sample from a skull at the time of death, how many milligrams of ^{14}C would be present in the skull after each of the following periods?

a. 5000 years

b. 25,000 years

c. 50,000 years

29. An archaeologist discovers a burial site that she believes to be 8000 years old. Examination of bones from the site shows that 40% of its ^{14}C is still present. Is the archeologist's belief reasonable? Justify your answer.

30. Charcoal from a suspected ancient campfire is tested and found to contain only 0.5% of the original amount of ^{14}C. Determine the age of the charcoal.

Extended Problems

31. Predictions about the future of the world's population have led to warnings about overpopulation and a lack of resources. Although the world's population is increasing, many individual countries are experiencing decreasing populations.

a. Use the Internet to research world population growth. List five countries with large rates of growth, and list several countries currently experiencing declining populations. Describe any geographic or economic similarities between the countries with fast-growing populations. What similarities can be found between countries with declining populations? Why might the countries with declining populations be concerned about their lack of growth? For information on the Internet, search keywords "world population growth."

b. Write an essay describing the factors often considered when researchers make predictions about the world's population. List several current predictions about the future of the world's population and explain why they differ. For information on the Internet use search keywords "world population predictions."

32. The Shroud of Turin is a piece of linen cloth that bears the negative image of the front and back of a man. The image is that of a man with a beard, long hair and a mustache, and wounds on his body, most of which are consistent with having been flogged and crucified. Many Christians believe that the cloth was the fabric in which Jesus was wrapped after his crucifixion. The cloth has been tested in an attempt to establish its age. Research the Shroud of Turin, and write a report about attempts to estimate its age. Include information about dating techniques that have been used and conclusions that have been drawn. On the Internet use search keywords "Shroud of Turin."

33. Carbon dating is only one of about 40 different radiometric methods used to date a sample. Research three of the other methods. Which methods are commonly used today? What assumptions must be made when relying on these methods? Which are thought to be the most accurate? According to these methods, how old is the earth? For information on the Internet use search keywords "radiometric dating techniques."

34. Techniques for dating fossils and bones are controversial. The use of carbon-14 to date organic objects requires several assumptions. If these assumptions cannot be made, then any dates obtained as a result of carbon-14 dating may not be accurate. Research carbon dating. Write a report that summarizes the following:

What items cannot be accurately dated using this technique? What are the ideal conditions under which carbon-14 dating can be used to estimate the age of a sample? What assumptions must be made in order to rely on the dates obtained using this method? On the Internet use search keywords "carbon dating controversy."