

Math 354, Class Exercise 10

Predator Prey Modeling

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1. (This is a closed book active learning exercise!) Consider a 2 species model of predator and prey. To simplify, let us consider the following two species in a closed surrounding:

F = number of a certain species of fish (eaten by sharks) like tuna, mackerel, or sardines for example in a specific region of the sea.

S = number of sharks in the same area

The area is assumed to be bounded with no migration across the boundary of the region. Then the rates of change of the population of the fish and sharks depends on the number of fish and sharks present. That is:

$$\frac{dF}{dt} = g(F, S) \tag{1}$$

$$\frac{dS}{dt} = h(F, S) \tag{2}$$

Under whatever assumptions that you see fit, write down a mathematical model for the rates of change of the population of the sharks and fish. Be logical about your assumptions. Some things to consider would be:

- I. The number of sharks and fish depend on each other. i.e. if there is a spike in the fish population, then the shark population should increase. If there is a sharp decline, then the shark population should decline as well.
 - II. Are the fish the only source of food for the sharks? If there are no fish and you start with an initial shark population, what will happen to them? What can you say explicitly about the rate of change of the shark population?
 - III. If we assume that food for the fish is very abundant, what kind of growth model would we initially assume on the fish? What if the food supply is limited but still abundant, then what can we say? Specific examples would be the abundance of plankton for sardines or crustaceans and small fin fish for mackerel or tuna.
 - IV. In what way does the fish population depend on the shark population? i.e. how does shark predation affect the fish population in regard to the mathematical modeling? Write this down explicitly.
 - V. In what way does the shark population depend on the fish population? Write this down explicitly.
2. What are some qualitative properties of your 2 species model? i.e. what are the equilibrium solutions and what do these correspond to in the real world?
 3. What kind of analysis can you do to further describe your system? Utilize these tools as much as possible to describe the behavior.
 4. How's about a three species ecosystem? Can you think of one and come up with an explicit model? What are the assumptions?