Math 354, Class Exercise 4
Logistic Equation
Active Learning Approach Instructor: Dr. Fred Park

1. Find an explicit solution of the logistic equation below and graph a solution for N_0 starting below the saturation level $N = \frac{a}{b}$

$$\frac{dN}{dt} = N(a - bN)$$

2. Consider $dN/dt = \alpha N^2 - \beta N$ with $\alpha, \beta > 0$.
   (a) How does the growth rate depend on the population?
   (b) Sketch the solution in the phase plane labeling all equilibria. Sketch typical solutions.
   (c) Are there any inflection points in the solution assuming sufficient smoothness of solutions?
   (d) Obtain the exact solution.
   (e) Show how parts (b) and (c) illustrate the following behavior:
      i. If $N_0 > \beta/\alpha$, then $N \to \infty$. At what time does $N \to \infty$?
      ii. If $N_0 < \beta/\alpha$, then $N \to 0$.
      iii. What happens if $N_0 = \beta/\alpha$?