Math 354, Class Exercise 5
Variations of the Logistic Equation and Least Squares Fitting
Instructor: Dr. Fred Park

1. **"Best fit Curve" of the Theoretical Population Model to Given Data**
   If some discrete population data $N_d(t_m)$ is known (not necessarily measured in equal time intervals). Then the mean-square deviation between the data and a theoretical curve $N(t) = N_0 e^{R_0 t}$ is the sum of the squared differences below:
   $$\sum_m [N(t_m) - N_d(t_m)]^2$$
   The "best fit" is often defined as those values that minimize the above mean-square deviation.

   (a) Assume that the initial population N_0 is known with complete certainty, so that we insist that the theoretical population curve initially agree exactly. Assume the theoretical curve exhibits exponential growth. By minimizing the above mean-square deviation, obtain an equation for the best estimate of the growth rate. Show that this is a transcendental equation.

   (b) One way to bypass the difficulty in part (a) is to fit the natural logarithm of the data to the natural logarithm of the theoretical curve. In this way, the mean square deviation is the following:
   $$\sum_m [\ln N_0 + R_0 t_m \ - \ \ln N_d(t_m)]^2$$
   Show that this method is now the least squares fit of a straight line to the data as outlined in class. If N_0 is known, (i.e. $N_0 = N_d(t_0)$), determine the best estimate of the growth rate using this criteria.

   (c) Redo part (c) above assuming now that a best estimate of the initial population is also now desired. i.e. minimize the mean-square deviation with respect to both N_0 and R_0.

2. **Matlab Computing: Fitting Noisy Population Data.**

   (a) Sample non-uniformly in time an exponential growth curve of your choice. Note, the non in non-uniform.

   (b) Randomly perturb the points so that the values shift up or down by a small amount by adding a small amount of noise to the data. The noise can be created by using the *randn* function in matlab. Use the help command if you are confused.

   (c) Use the above model in part (b) to fit in log-space a linear best fit to your given data.

   (d) Can you think of a way to map back to data that better resembles an exponential growth curve?

   (e) You will test your code on the data set obtained on my website called:"gen_growth_data.m"

3. **Matlab Computing: Logistic Equation Revisited.**
   Sketch the solution N to the Logistic Equation as a function of t by using the direction field and the method of isoclines.