Math 354, Class Exercise 7
Solving Linearized Systems of DE's
Instructor: Dr. Fred Park

1. Expand the non-linear system directly using Taylor Expansion around an equilibrium solution. Do you obtain the same answer as the method in class?

2. Use the ansatz $y = e^{rt}$ into the following system

$$\frac{dx}{dt} = ax + by \tag{1}$$
$$\frac{dy}{dt} = cx + dy \tag{2}$$

   obtain the appropriate values for r. Then write out the solutions for y as $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$. How do you find the solution x?

3. (a) Change the system to a matrix form. Let A be the coefficient matrix. Let

$$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

   Then

$$\frac{d\vec{v}}{dt} = A\vec{v}$$

   (b) Assume solutions are of the form $\vec{v} = \vec{v}_0 e^{rt}$ and substitute into the system. What do you see?

   (c) If we assume non-trivial solutions (i.e. $\vec{v}_0 \neq \vec{0}$), can you change the problem into a matrix one?

   (d) Can you find vectors $\vec{v}_1$ and $\vec{v}_2$ and values $r_1,\ r_2$ so that solutions $\vec{v}$ are of the form:

$$\vec{v} = c_1 \vec{v_1} e^{r_1 t} + c_2 \vec{v_2} e^{r_2 t}$$

4. Consider the system:

$$\frac{dx}{dt} = ax + by \tag{3}$$
$$\frac{dy}{dt} = cx + dy \tag{4}$$

   Find solutions in terms of time t in the following cases:

   (a) a=0, b=1, c=-4, d=0

   (b) a=1, b= 2, c=-1, d=2

   (c) a=-1, b=3, c=1, d=1

   (d) a=2, b= -1, c=1, d=0