

# COSC 120: Class Exercise 4

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### 1. Application of root-finding:

Here, we will look at the cannon problem. Suppose you fire a canon and a projectile is launched with speed  $v$  and angle  $\theta$ .

Question: What angle  $\theta$  should we use to force our canon to hit a target at a distance  $R$  away? Please note, you may make the problem a more peaceful and fun one by allowing the projectile to be a water balloon and the target can be a group of your fellow students engaged in a scientific water-balloon fight with partially protective safety shelters for each group.

The trajectory of the water balloon is given by

$$x(t) = vt \cos \theta \quad (1)$$

$$y(t) = vt \sin \theta - \frac{1}{2}gt^2 \quad (2)$$

here,  $g = 9.8 \text{ m/s}^2$ .

- a From above, solve for  $y$  as a function of  $x$ , i.e.  $y = y(x)$ .
- b Once you have  $y = y(x)$ , the range  $R$  is the location when  $y = 0$ . Why? This is precisely now a root-finding problem. If  $\theta = \pi/4$  and  $v = 40 \text{ m/s}$ . If the opposing water balloon team is located 150 meters away, will you be able to hit them with a water balloon? Use your bisection method code to find a solution to this important problem.
- c What angle  $\theta$  will allow you to hit them? Here, you will adjust  $\theta$  in your code so that you can hit your target.
- d How fast does the muzzle speed need to be to hit a target 1000 meters away assuming that  $\theta = \pi/4$ ? Use your code to find this.

### 2. The Monty Hall Problem:

“Suppose you’re on a game show, and you’re given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what’s behind the doors, opens another door, say No. 3, which has a goat. He then says to you, ”Do you want to pick door No. 2?” Is it to your advantage to switch your choice?”

Calculate the probability if you choose a strategy where you always switch. Do the same for if you always stay with your initial choice. Is it feasible to always stay or to always switch?

Write a simulation in Python for the Monty Hall problem using 10 million runs for the case where you always switch. What is the result? Does it agree with your calculation of the theoretical probability? Here, you will need to use the ‘random’ module and the ‘randint(a,b)’ function from this module which generates a random integer from a to b inclusively. Note: you can type ‘help(random)’ to obtain a list of the contents of the random module.