Math 242 Midterm #2

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Key!

Name:______________________________

Student ID#:______________________________

Note: For full credit you must show all work. Incoherent work without logic or reason will not receive any credit whatsoever! Please circle your final answers.

1. (25 points) Find the inverse of the following matrix if it exists. If it does not, state explicitly why.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix} \tag{1}$$

2. (25 points) Do as indicated:

   (a) Combine the methods of row reduction and cofactor expansion to calculate the following determinant:

$$\begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{vmatrix} \tag{2}$$

   (b) Show that if a matrix A is invertible, then

$$\det A^{-1} = \frac{1}{\det A}$$

3. (25 points) Do as indicated.

   (a) Find a basis for both the null space and column space of the matrix:

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 4 & 3 & 5 \\ -2 & 2 & -6 \\ -5 & 4 & -14 \end{bmatrix} \tag{3}$$

   (b) If we consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ given by $T(\vec{v}) = A\vec{v}$. Using the matrix in part (a), is the transformation T onto? Is it one-to-one? Why or why not?

4. (25 points) Do this problem last!!! Consider the polynomials $\mathbf{p}_1(t) = 1 + t$, $\mathbf{p}_2(t) = 1 - t$, and $\mathbf{p}_3(t) = 2$. Find a basis for Span $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$

① $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}$

$A^{-1} = ?$

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 3 & 0 & 1 & 0 \\ 4 & -3 & 8 & 0 & 0 & 1 \end{array}\right) \xrightarrow{RREF}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -9/2 & 7 & -3/2 \\ 0 & 1 & 0 & -2 & 4 & -1 \\ 0 & 0 & 1 & 3/2 & -2 & 1/2 \end{array}\right)$$

$$\underbrace{\begin{matrix} -9/2 & 7 & -3/2 \\ -2 & 4 & -1 \\ 3/2 & -2 & 1/2 \end{matrix}}_{A^{-1}}$$

(29)

$$A = \begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{vmatrix} = ?$$

$R_3 \to -2R_4 + R_3$

$$B = \begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ -3 & 0 & -2 & 0 \\ 4 & 2 & 4 & 3 \end{vmatrix} = 3(-1)^{4+4} \det B_{44}$$

$$= 3 \begin{vmatrix} -1 & 2 & 3 \\ 3 & 4 & 3 \\ -3 & 0 & -2 \end{vmatrix}$$

$R_2 \to -2R_1 + R_2$

$$= 3 \begin{vmatrix} -1 & 2 & 3 \\ 5 & 0 & -3 \\ -3 & 0 & -2 \end{vmatrix} = 3(2)(-1)^{1+2} \begin{vmatrix} 5 & -3 \\ -3 & -2 \end{vmatrix}$$

$$= 114$$

26 show if A invertible

then $\det(A^{-1}) = \dfrac{1}{\det(A)}$.

A invertible $\to \exists\, B \ni$

$AB = BA = I_n$

$BA = I_n \to \det(BA) = \det(I_n) = 1$

$\to \det(B)\det(A) = 1$

$\to \det(B) = \dfrac{1}{\det(A)}$

but $B = A^{-1}$

$\to \det(A^{-1}) = \dfrac{1}{\det(A)}$.

3a $A = \begin{pmatrix} 3 & 4 & 2 \\ 4 & 3 & 5 \\ -2 & 2 & -6 \\ -5 & 4 & -14 \end{pmatrix}$

RREF $\rightarrow \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow$ $x_1 = -2x_3$

$x_2 = x_3$

$x_3 = x_3$

$\rightarrow$ $x_1 = -2t$

$x_2 = t$ $\quad \therefore \quad \vec{x} = t \begin{pmatrix} -2 \\ 1 \\ \phi \end{pmatrix}$

$x_3 = t$

$\rightarrow \operatorname{Nul}(A) = \operatorname{span}\left\{ \begin{pmatrix} -2 \\ 1 \\ \phi \end{pmatrix} \right\}$.

pivot positions in column 1 & 2

$\rightarrow$ col 1 & col 2 of A are L.I.

$\therefore$

$\operatorname{col}(A) = \operatorname{span}\left\{ \begin{pmatrix} 3 \\ 4 \\ -2 \\ 5 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ 2 \\ 4 \end{pmatrix} \right\}$.

(3b) Let $T(\vec{v}) = A\vec{v}$

Then $T(\vec{v}) = \vec{0}$ w/ $\vec{v} \neq \vec{0}$

Let $\vec{v} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

$\therefore$ not 1-1.

Not onto. $T(\vec{v}) = A\vec{v}$

$T: \mathbb{R}^3 \to \mathbb{R}^4$

$\mathrm{col}(A) \neq \mathbb{R}^4$.

Let $\vec{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \in \mathbb{R}^4$

Then $\vec{v} \notin \mathrm{span}\left\{ \begin{pmatrix} 3 \\ 4 \\ -2 \\ 5 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ 2 \\ 4 \end{pmatrix} \right\}$

since

$$\left[\begin{array}{cc|c} 3 & 4 & 0 \\ 4 & 3 & 0 \\ -2 & 2 & 0 \\ 5 & 4 & 1 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right] \to 0 = 1$$

$\times$.

$\to$ not onto.

(4) $p_1(t) = 1+t$

$p_2(t) = 1-t$

$p_3(t) = 2$

basis for span $\{p_1, p_2, p_3\}$?

note: $p_3(t) = p_1 + p_2 = 1+t+1-t = 2$

$\therefore$ $p_3 \in \text{span}\{p_1, p_2\}$.

$\therefore$ $\text{span}\{p_1, p_2, p_3\} = \text{span}\{p_1, p_2\}$.

if p_1 & p_2 Lin. Indep.

then $\{p_1, p_2\}$ basis for Span$\{p_1, p_2, p_3\}$.

let $a, b \in \mathbb{R}$

$a p_1 + b p_2 = 0$

$\rightarrow a(1+t) + b(1-t) = 0$

$\rightarrow (a+b)\cdot 1 + (a-b)t = 0$

$\rightarrow \left.\begin{array}{l} a = -b \\ \& \; a = b \end{array}\right\}$ $\rightarrow 2a = 0$

$\rightarrow a = 0$

$\rightarrow b = 0$

$\rightarrow$ p_1 & p_2 Lin Indep. $\square$.