Math 141A Practice Final
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Note: this exam is only slightly longer than the actual final

1. Find the area of the largest rectangle that can be inscribed inside a semi-circle of radius r.
2. Find correct to 6 decimal places the root of the equation $\cos x = x$.
3. Evaluate the Limits
   - $\lim_{x\to 0}(a^x - 1)/x$ $(a > 0)$
   - $\lim_{x\to 0} x^3/(\sin x - x)$
   - $\lim_{x\to 0^+} x^{\sin x}$
   - $\lim_{x\to\infty} x^{1/x^2}$
   - $\lim_{x\to\pi/2} \tan 4x/\tan 5x$
   - $\lim_{x\to\infty} \ln x/x^{1/2}$
4. Sketch the Graph of $y = (2x^3 + x^2 + 1)/(x^2 + 1)$
5. Find the maximum area of a rectangle inscribed in the region bounded by the graph $y = (4 - x)/(2 + x)$.
6. Find the max area of a triangle formed by the axes and the tangent to the graph $y = (x+1)^{-2}$.
7. A rocket is traveling upwards at a speed of 1200km/h. The rocket is tracked through a telescope by an observer located 16km from the launching pad. Find the rate at which the angle between the telescope and the ground is increasing 3mins after liftoff.
8. Use linear approximation and a calculator to estimate the error $625^{1/4} - 624^{1/4}$
9. Sketch the graph $y = 12x - 3x^2$
10. Use Newton's method to estimate $25^{1/3}$ to four decimal places
11. (a) Calculate $\int(4x^3 - 2x^2)dx$
    (b) Calculate $\int \sin(\theta - 8)d\theta$
    (c) Calculate $\int(x + 2)^4 dx$
    (d) Calculate $\int e^{2x}\sin(3x)dx$
    (e) Calculate $\int x^3 e^{x^2} dx$
    (f) Calculate $\int 1/(x - 2)(x + 2)dx$
    (g) Calculate $\int z^2 e^z dz$
    (h) Calculate $\int t^4 \ln t dt$
    (i) Calculate $\int 1/(x^2 + 4x + 5)dx$ (hint:complete the square)
12. Show the phase plane diagram, typical solutions for $dy/dx = -(y - 2)(y + 2)$. Solve the Differential equation by separation of variables.
13. Newton's law of cooling says that if $T = T(t)$ is the temperature of an object placed in a surrounding environment with ambient temperature T_{amb}. Then the ROC of T is proportional to the difference between the ambient temp and the temp of the object. Thus

$$\frac{dT}{dt} = k(T_{amb} - T), \quad (k > 0)$$

If we submerge a hot metal bar into a tank of water of ambient temp $T_{amb} = 10$ degrees C. If the cooling constant $k = 2.1\text{min}^{-1}$. Find the bar's temp after 1 min if its initial temp was 180 degrees C. What was the bar's initial temp if it cooled to 80 degrees C in 30 s?