Math 241: Practice Midterm #2

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Note: This Exam is Slightly Longer than the Actual Exam

1. Find the equation of the tangent plane and the normal line to the surface given by $xy + yz + zx = 3$ at the point $(1, 1, 1)$.

2. Find the points on the sphere $x^2+y^2+z^2 = 1$ where the tangent line is parallel to the plane $2x + y - 3z = 2$.

3. Use the chain rule to find $\partial w/\partial t$ where $w = \sqrt{x} + (y^2/z)$, $y = t^3+4t$, $x = e^{2t}$, and $z = t^2 - 4$.

4. Use a tree diagram to write out the chain rule for the case where $w = f(t, u, v)$, $t = t(p, q, r, s)$, $u = u(p, q, r, s)$, and $v = v(p, q, r, s)$ are all differentiable functions.

5. The length x of a side of a triangle is increasing at a rate of 3 in/s, the length y of another side is decreasing at a rate of 2 in/s, and the contained angle θ is increasing at a rate of 0.05 radians/s. How fast is the area of the triangle changing when $x = 40$ in, $y = 50$ in, and $\theta = \pi/6$?

6. If $yz^4 + x^2z^3 = e^{xyz}$, find $\partial z/\partial x$ and $\partial z/\partial y$.

7. Directional Derivative. Let $f = f(x, y, z)$ be a differentiable function:

   (a) When is the directional derivative of f a maximum?

   (b) When is it a minimum?

   (c) When is it zero?

   (d) When is it half its maximum value?

8. Find the directional derivative of $f(x, y) = 2\sqrt{x} - y^2$ at the point $(1, 5)$ in the direction toward the point $(4, 1)$.

9. Find the maximum rate of change of $f(x, y) = x^2y+\sqrt{y}$ at the point (2,1). In which direction does this maximum ROC take place?

10. Find the local maximum and minimum values and saddle points of the function $f(x, y) = x^2 - xy + y^2 + 9x - 6y + 10$.

11. Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = xyz$ subject to the constraint $x^2 + y^2 + z^2 = 3$.

12. Find the points on the surface $xy^2z^3 = 2$ that are closest to the origin.

13. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.

14. Use either multivariable Optimization or Lagrange multipliers to find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the general ellipsoid given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

15. Calculate $\iiint_E x^2 dV$ where E is the tetrahedron with vertices $(0,0,0), (1,0,0), (1,1,0), (1,0,1)$.

16. Show $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{x^2 + y^2 + z^2}\, e^{-(x^2+y^2+z^2)} dxdydz = 2\pi$.